Chapter 17

Gear Forces

THE COMPONENTS OF GEAR FORCES are usually determined rather than the resultant gear force, although the latter can be found by the vector sum of the components. The components are used in calculating bearing reactions, shaft size, etc.

FRICTION LOSSES in spur, helical, and bevel gears are usually so small that these gears are considered as operating at 100% efficiency. There are situations where the friction in spur gearing, even though low, must be taken into account, as in the case of circulating power in planetary gearing.

Worm and worm gears, however, are usually not as efficient as spur, bevel, and helical gears; hence friction is usually taken into account in determining the force components on worm and worm gears.

SPUR GEAR force components are (see Fig. 17-1 below)

(1) Tangential force $F_t = M_t/r$, where M_t = gear torque and r = pitch radius of the gear.

(2) Separating or radial force $F_r = F_t \tan\phi$ where ϕ is the pressure angle.
Note that the radial force is always directed towards the center of the gear.

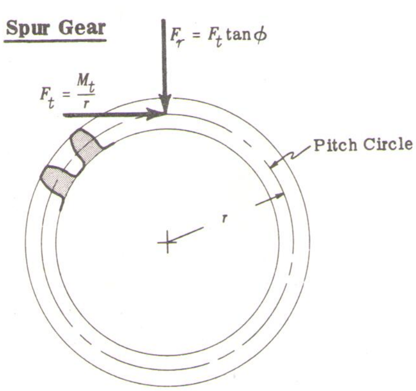


Fig. 17-1

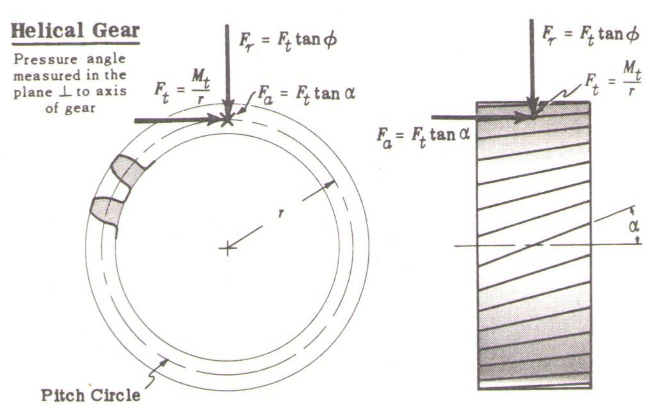


Fig. 17-2

HELICAL GEAR force components are expressed in two different ways depending upon how the pressure angle is defined. There are two standards: (1) the pressure angle ϕ is measured in the plane perpendicular to the axis of the gear and (2) the pressure angle ϕ_n is measured in a plane normal to a tooth. See Fig. 17-2 above and Fig. 17-3 below.

(1) If the pressure angle is measured in a plane perpendicular to the axis of the gear, the components are (see Fig. 17-2 above):

a. Tangential force $F_t = M_t/r$

b. Separating force $F_r = F_t \tan \phi$

c. Thrust force $F_a = F_t \tan \alpha$

where r = gear pitch radius,

ϕ = pressure angle measured in a plane perpendicular to the axis of the gear,

α = helix angle measured from the axis of the gear.

(2) If the pressure angle is measured in a plane perpendicular to a tooth, the components are (see Fig. 17-3):

a. Tangential force $F_t = M_t/r$

b. Separating force $F_r = \dfrac{F_t \tan \phi_n}{\cos \alpha}$

c. Thrust force $F_a = F_t \tan \alpha$

where ϕ_n = pressure angle measured in a plane perpendicular to a tooth,

α = helix angle measured from the axis of the gear.

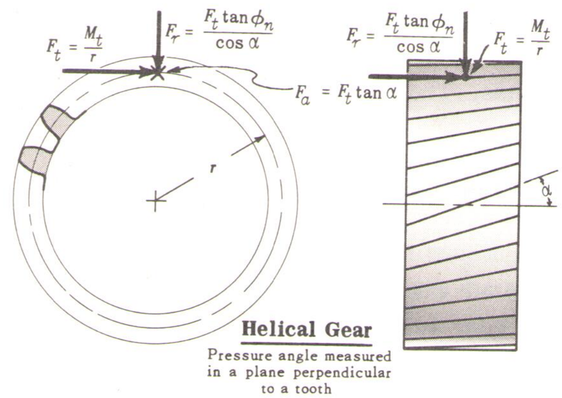


Fig. 17-3

The direction of the thrust force depends on the direction of rotation and the hand of the gear teeth. Four possibilities of combinations of right and left hand helical gears with different combinations of rotation are shown in Fig. 17-4 below, with the direction of thrust. Reversing the direction of rotation of the driver will reverse the direction of thrust from that shown.

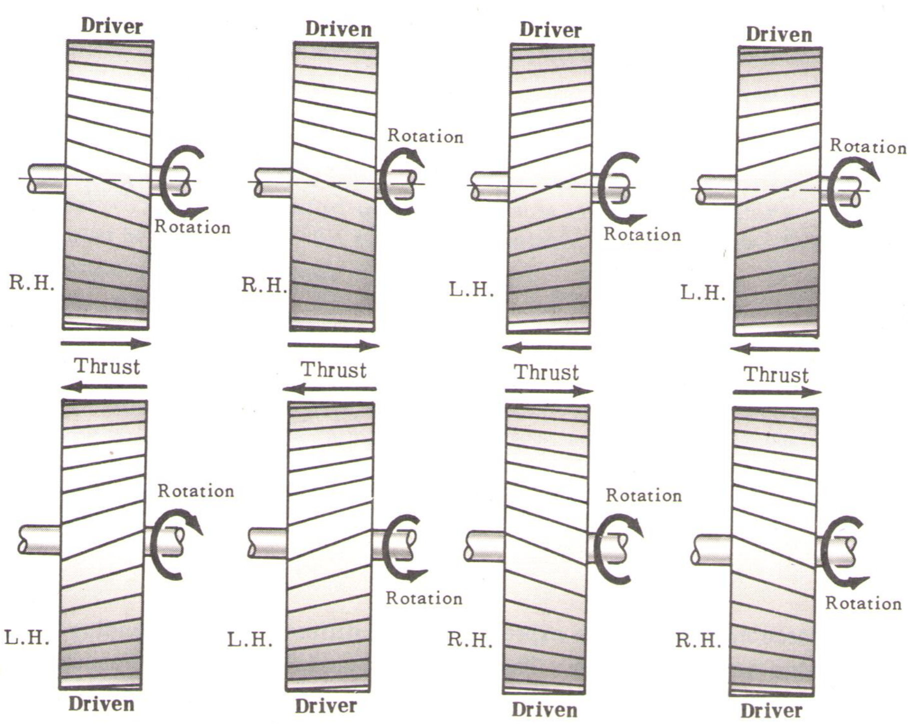


Fig. 17-4

STRAIGHT TOOTH BEVEL GEAR force components, shown in Fig. 17-5(*a*) below, are:

(1) Tangential force $F_t = M_t/r$.
This force is considered acting at the mean pitch radius r.

(2) Separating force $F_r = F_t \tan\phi$
where ϕ is the pressure angle. The separating force can be resolved into two components; the force component along the shaft axis of the pinion is called the pinion thrust force F_p, and the force component along the shaft axis of the gear is called the gear thrust force F_g.

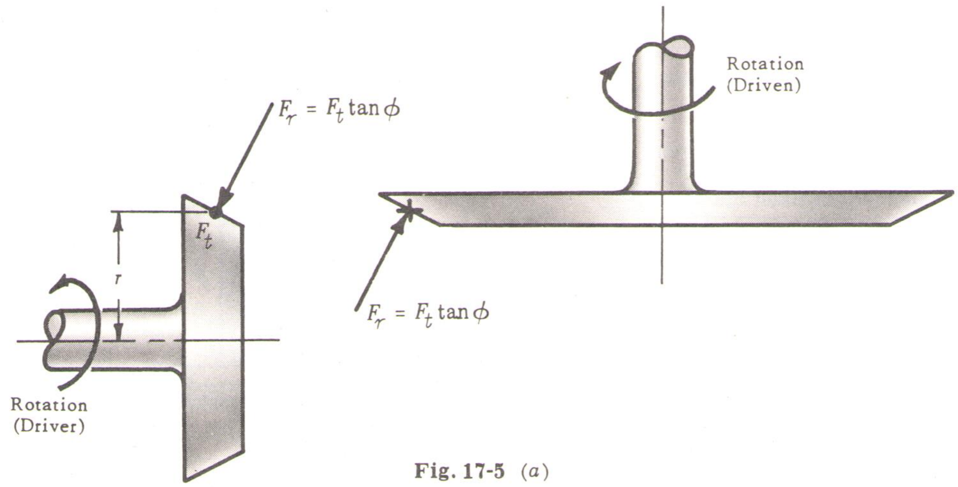


Fig. 17-5 (*a*)

The three mutually perpendicular components, shown in Fig. 17-5(*b*) below, are:

a. The tangential force $F_t = M_t/r$ acting at the mean pinion pitch radius r, where M_t is the pinion torque.

b. The pinion thrust force $F_p = F_t \tan\phi \sin\beta$, where β is the pitch cone angle of the pinion.

c. The gear thrust force $F_g = F_t \tan\phi \cos\beta$.

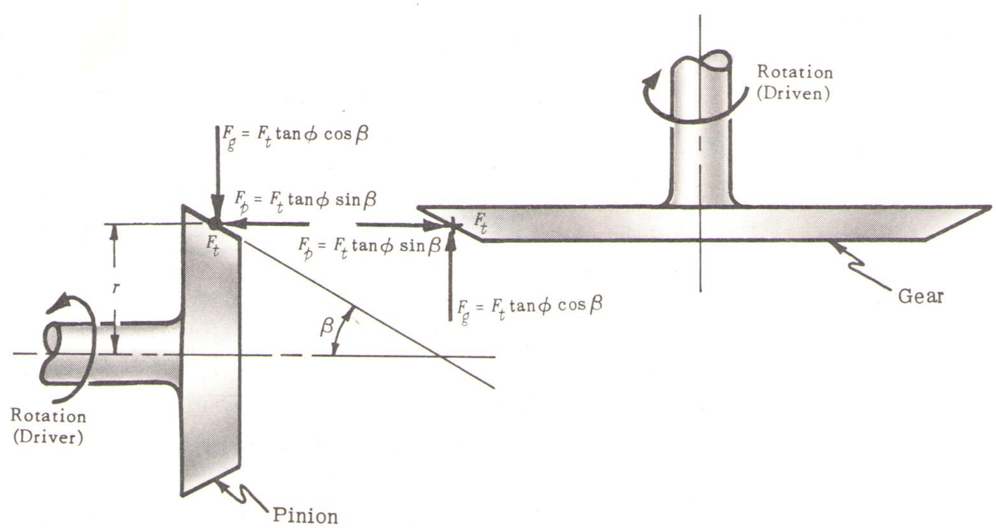


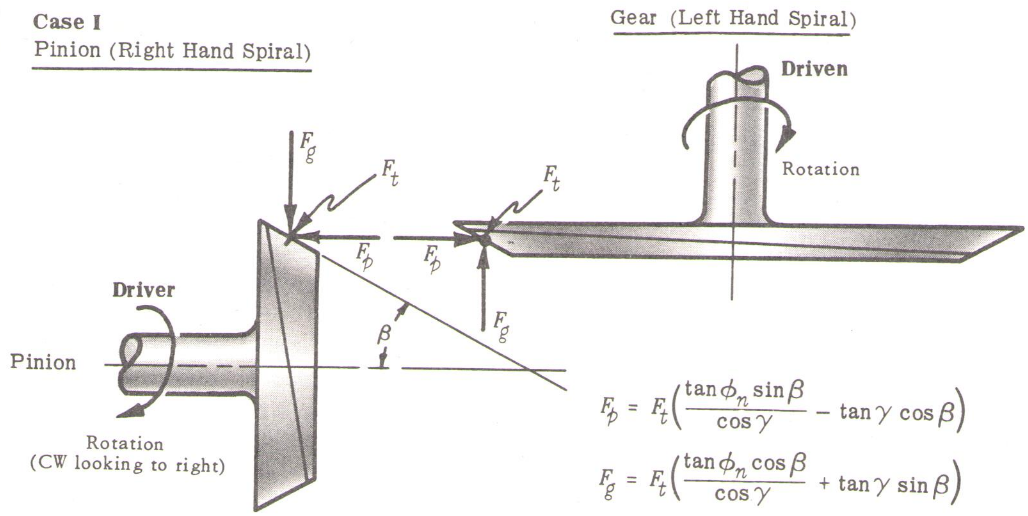
Fig. 17-5 (*b*)

SPIRAL BEVEL GEAR force components are more involved than those for a straight tooth bevel gear.

The tangential force at the mean pitch radius r is $F_t = M_t/r$, where M_t is the torque.

The pinion thrust force F_p and the gear thrust force F_g can be expressed in different ways, depending on how the pressure angle is measured. Pinion and gear thrust forces, with the pressure angle ϕ_n measured in the plane normal to the tooth are shown in Fig. 17-6 (*a* to *d*) for different hand of spirals (that is, left hand and right hand) and for different directions of rotation. The symbols are:

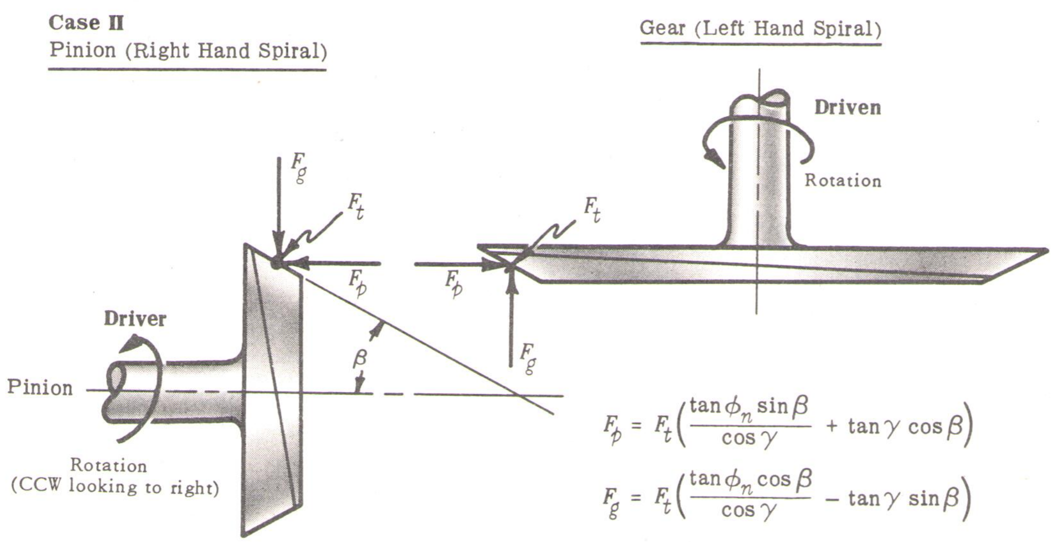
F_p = pinion thrust force
F_g = gear thrust force
F_t = tangential force causing the torque at the mean radius r
ϕ_n = tooth pressure angle measured in the plane normal to a tooth
β = pinion pitch cone angle
γ = pinion spiral angle.



$$F_p = F_t\left(\frac{\tan\phi_n \sin\beta}{\cos\gamma} - \tan\gamma\cos\beta\right)$$

$$F_g = F_t\left(\frac{\tan\phi_n \cos\beta}{\cos\gamma} + \tan\gamma\sin\beta\right)$$

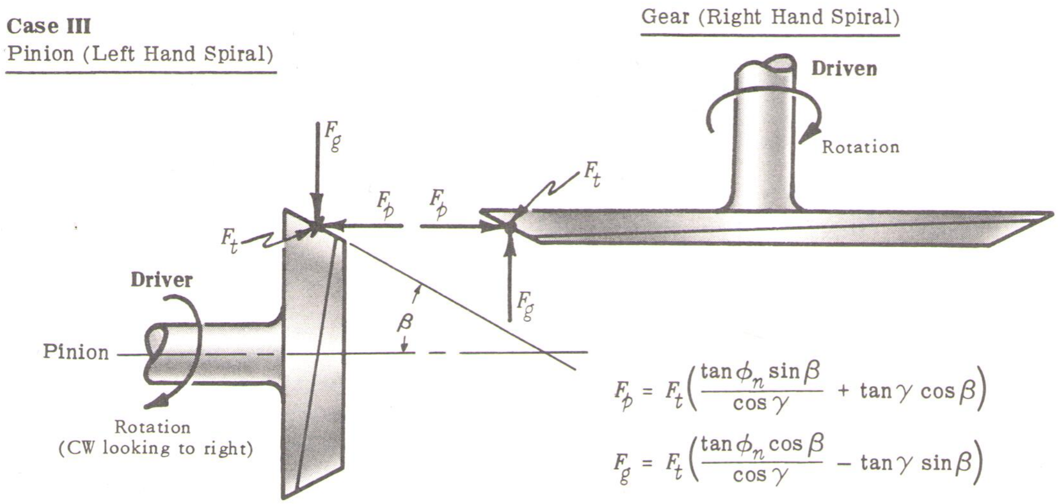
Fig. 17-6 (*a*)



$$F_p = F_t\left(\frac{\tan\phi_n \sin\beta}{\cos\gamma} + \tan\gamma\cos\beta\right)$$

$$F_g = F_t\left(\frac{\tan\phi_n \cos\beta}{\cos\gamma} - \tan\gamma\sin\beta\right)$$

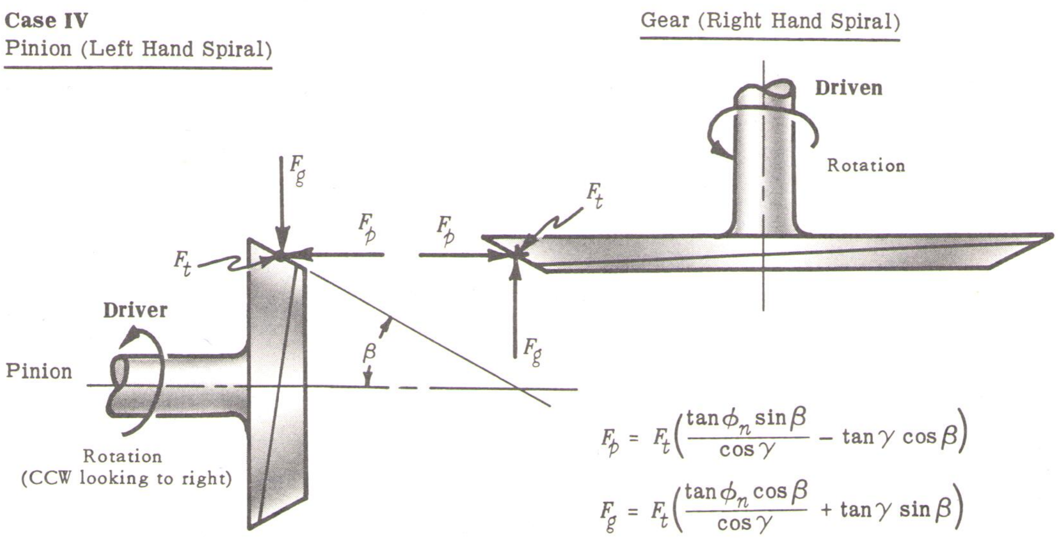
Fig. 17-6 (*b*)



$$F_p = F_t\left(\frac{\tan\phi_n \sin\beta}{\cos\gamma} + \tan\gamma\cos\beta\right)$$

$$F_g = F_t\left(\frac{\tan\phi_n \cos\beta}{\cos\gamma} - \tan\gamma\sin\beta\right)$$

Fig. 17-6(*c*)



$$F_p = F_t\left(\frac{\tan\phi_n \sin\beta}{\cos\gamma} - \tan\gamma\cos\beta\right)$$

$$F_g = F_t\left(\frac{\tan\phi_n \cos\beta}{\cos\gamma} + \tan\gamma\sin\beta\right)$$

Fig. 17-6(*d*)

The spiral angle is measured as shown in Fig. 17-7.

If the forces are found to be positive, they are directed as shown in the figure; if negative, they are opposite in direction to that shown.

If the pressure angle ϕ is measured in a plane normal to a pitch cone element, the equations given with the figure are changed by substituting

$$\tan\phi = \frac{\tan\phi_n}{\cos\gamma}$$

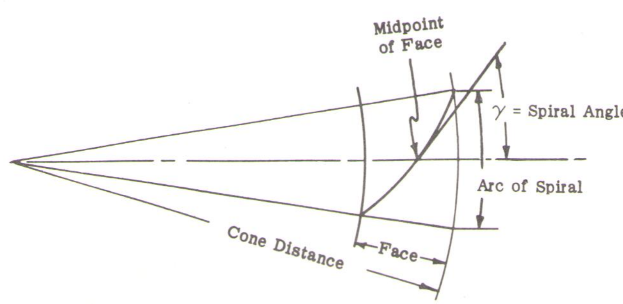


Fig. 17-7

WORM GEARING. The three mutually perpendicular components of the resultant force acting between a worm and worm gear are:

(1) $F_{t(\text{worm})} = M_t/r_w$ where $F_{t(\text{worm})}$ = tangential force on the worm
M_t = torque on the worm
r_w = pitch radius of the worm.

(2) $$F_{t(\text{gear})} = F_{t(\text{worm})}\left(\frac{1 - f\tan\alpha/\cos\phi_n}{\tan\alpha + f/\cos\phi_n}\right)$$

where $F_{t(\text{gear})}$ = tangential force on the worm gear
f = coefficient of friction
α = lead angle of the worm (which is the same as the helix angle of the worm gear). The lead angle of the worm is found from $\tan\alpha = \text{lead}/(\pi D_w)$, where the lead is the number of threads times the linear pitch of the worm and D_w is the pitch diameter of the worm. Note that the linear pitch of the worm is equal to the circular pitch of the worm gear.
ϕ_n = normal pressure angle measured in a plane perpendicular to a tooth (usually $14\frac{1}{2}°$ for a single or double thread and $20°$ for a triple or quadruple thread).

(3) $$F_r = F_{t(\text{gear})}\left(\frac{\sin\phi_n}{\cos\phi_n\cos\alpha - f\sin\alpha}\right) = F_{t(\text{worm})}\left(\frac{\sin\phi_n}{\cos\phi_n\sin\alpha + f\cos\alpha}\right)$$

where F_r is the separating force.

Fig. 17-8 below, shows the forces for different directions of rotation and hand of worm threads.

The pressure angle ϕ measured in the plane containing the axis of the worm is related to the pressure angle ϕ_n measured in a plane normal to a worm thread by $\tan\phi_n/\cos\alpha$.

Case I
Driver: Worm (Right Hand)

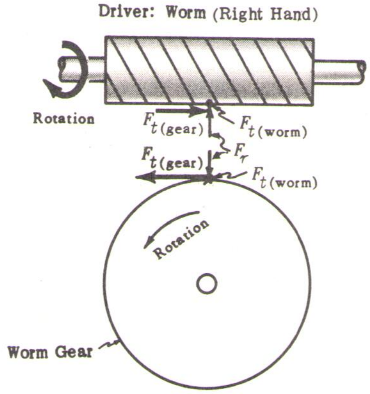


Fig. 17-8 (*a*)

Case II
Driver: Worm (Right Hand)

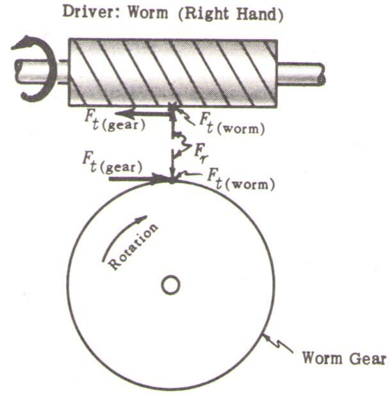


Fig. 17-8 (*b*)

Case III
Driver: Worm (Left Hand)

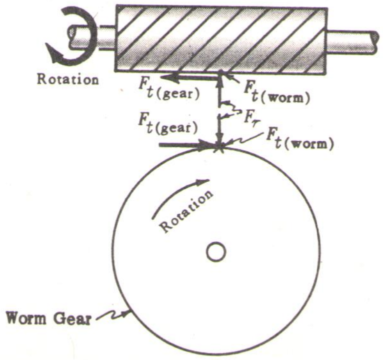


Fig. 17-8 (*c*)

Case IV
Driver: Worm (Left Hand)

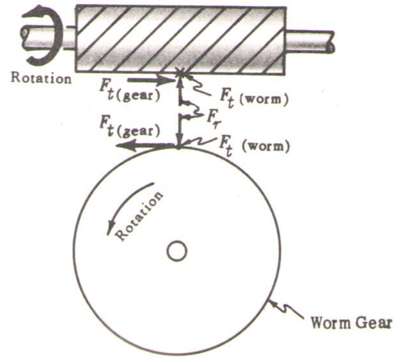


Fig. 17-8 (*d*)

THE FORCES IN PLANETARY GEAR TRAINS are obtained by the application of the basic equations of mechanics. These are illustrated in solved problems.

A problem encountered in planetary gearing with branch control circuits is the circulating power, which might be less or greater than the power input. The design of such a system can be simplified by the application of appropriate equations. Fig. 17-9 below, shows an arbitrary planetary gear system with a branch control circuit.

The circulating power ratio is given by

$$\gamma = \frac{r(1-R)}{1-r}$$

where $r = \omega_2/\omega_1$, $R = \omega_1/\omega_3$; and ω_1, ω_2 and ω_3 are respectively the angular velocities of elements 1, 2 and 3 as defined below.

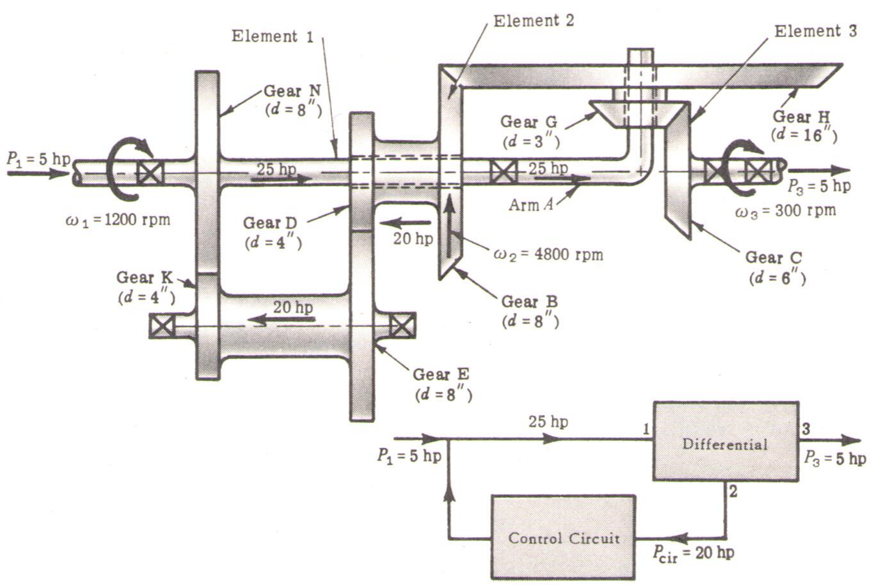


Fig. 17-9

Isolate the three basic elements of the planetary gear part, as shown in Fig. 17-10.

Element 3 is defined as that rotating element projecting from the differential directly to outside the system (gear C in the illustrative example) which has no connection with the control circuit. In some cases element 3 will be the arm; in other cases it may be one of the two gears which project from the differential.

Element 1 will always be that element, projecting from the differential to outside the system, that is connected to rotating element 2 by means of the branch control circuit.

Element 2 will always be that member which transmits power to or from the differential from or to the branch control circuit, but does not transmit power directly to or from the outside of the system. Thus, gear C is element 3, gear B is element 2, and the arm is element 1 for the example chosen.

The circulating power, P_{cir}, is

$$P_{cir} = \gamma P_3$$

where γ is defined above, and P_3 is the power through element 3.

The circulating power is the power in the branch control circuit, element 2.

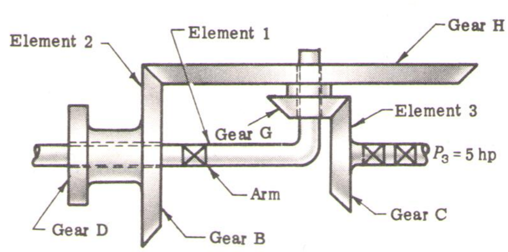


Fig. 17-10

SOLVED PROBLEMS

1. A spur pinion 4 in. in diameter has a torque of 2000 in-lb applied to it. The spur gear in mesh with it is 10 in. in diameter. The pressure angle is 20°. Determine the tangential force F_t and the separating force F_r, and show in position.

Solution:

$$F_t = M_t/r = 2000/2 = 1000 \text{ lb}$$

$$F_r = F_t \tan\phi = 1000 \tan 20^\circ = 364 \text{ lb}$$

The forces are shown in Fig. 17-11. Note that the tangential force on the pinion causes a torque to balance the applied torque, and the pinion separating force is directed towards the center of the pinion.

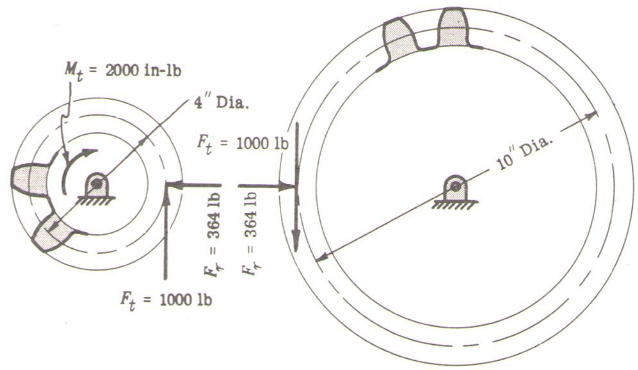


Fig. 17-11

2. Referring to Fig. 17-12, spur gear A receives 4 hp at 600 rpm through its shaft and rotates clockwise. Gear B is an idler and gear C is the driven gear. The teeth are 20° full depth. (The pitch circles are shown in the sketch.) Determine (*1*) the torque each shaft must transmit, (*2*) the tooth load for which each gear must be designed, (*3*) the force applied to the idler shaft as a result of the gear tooth loads.

Solution:

(*a*) Gear diameters: $D_A = 35/4 = 8\frac{3}{4}$ in.
$D_B = 65/4 = 16\frac{1}{4}$ in.
$D_C = 45/4 = 11\frac{1}{4}$ in.

(*b*) Torque on shaft of gear A
= (hp)(63,024)/N = 4(63,024)/600
= 420 in-lb.

Torque on shaft of gear B = 0.

Torque on shaft of gear C
= 4(63,024)/600(35/45) = 540 in-lb,
where gear C rotates at 600(35/45) rpm.

(*c*) Tangential force on gear A

$$= F_t = \frac{M_t}{r} = \frac{420}{\frac{1}{2}(8\frac{3}{4})} = 96 \text{ lb.}$$

Separating force on gear A

$$= F_t \tan\phi = 96 \tan 20^\circ = 35 \text{ lb.}$$

(*d*) The same tangential force and separating force occur between gears A and B and between gears B and C, in the direction shown.

(*e*) The tooth load for which each gear must be designed is 96 lb.

(*f*) The force applied to the idler shaft of gear B is the vector sum of the forces applied to gear B by gears A and C:

$$F_B = \sqrt{(96+35)^2 + (96+35)^2} = 185 \text{ lb.}$$

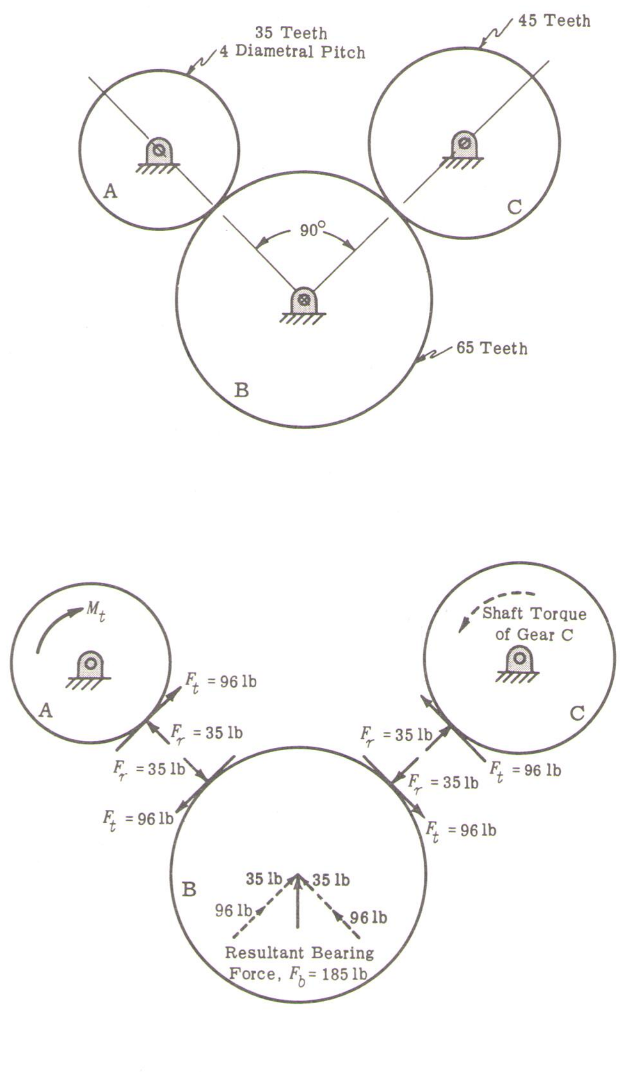


Fig. 17-12

3. A helical gear 9.00 in. in diameter has applied to it through its shaft a torque of 1800 in-lb. There are 45 teeth on the gear. The pressure angle, measured in a plane perpendicular to the axis of the gear, is 20°. The helix angle is 30°. Determine (*a*) the tangential force component F_t, (*b*) the separating force component F_r, (*c*) the axial thrust force component F_a. The helical gear has left hand teeth, and meshes with a right handed gear whose axis is directly above the axis of the left hand gear. Refer to Fig.17-13.

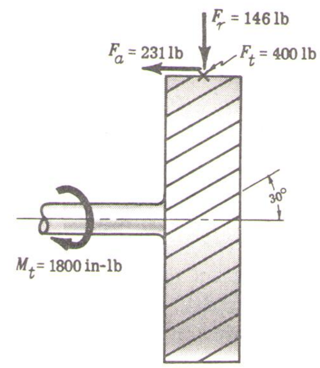


Fig. 17-13

Solution:

(*a*) $F_t = M_t/r = 1800/4.5 = 400$ lb

(*b*) $F_r = F_t \tan\phi = 400 \tan 20^\circ = 146$ lb

(*c*) $F_a = F_t \tan\alpha = 400 \tan 30^\circ = 231$ lb

The directions are as shown in Fig.17-13.

4. Repeat Problem 3, except that the pressure angle, measured in a plane perpendicular to the tooth, is 20°. Refer to Fig.17-14.

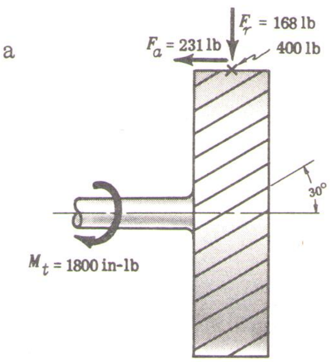


Fig. 17-14

Solution:

(*a*) $F_t = M_t/r = 1800/4.5 = 400$ lb

(*b*) $F_r = \dfrac{F_t \tan\phi_n}{\cos\alpha} = \dfrac{400 \tan 20^\circ}{\cos 30^\circ} = 168$ lb

(*c*) $F_a = F_t \tan\alpha = 400 \tan 30^\circ = 231$ lb

5. In Fig.17-15 a pair of straight tooth bevel gears has a velocity ratio of 4/3. The pitch diameter of the pinion is 6 in. The face width is $1\frac{1}{2}$ in. The pinion rotates at 240 rpm. The teeth are 5 diametral pitch, $14\frac{1}{2}^\circ$ involute. If 8 hp is transmitted, determine (*1*) the tangential force F_t at the mean radius, (*2*) the pinion thrust force F_p, (*3*) the gear thrust force F_g.

Solution:

(*a*) Diameter of the gear = 6(4/3) = 8 in.

(*b*) Slant height L of the pitch cone

$= \sqrt{R_p^2 + R_g^2} = \sqrt{3^2 + 4^2} = 5$ in.

(*c*) Mean radius of the pinion,

$r_m = R_p - \frac{1}{2}b \sin\beta = 3 - \frac{1}{2}(3/2)(3/5) = 2.55$ in.

(*d*) Pinion torque M_t

$= (\text{hp})(63{,}024)/N = 8(63{,}024)/240 = 2100$ in-lb.

(*e*) Tangential force at the mean radius, F_t

$= M_t/r_m = 2100/2.55 = 825$ lb.

(*f*) Pinion thrust force F_p

$= F_t \tan\phi \sin\beta = 825(\tan 14\frac{1}{2}^\circ)(3/5) = 129$ lb.

(*g*) Gear thrust force F_g

$= F_t \tan\phi \cos\beta$

$= 825(\tan 14\frac{1}{2}^\circ)(4/5) = 171$ lb.

The forces are shown in the free body diagrams.

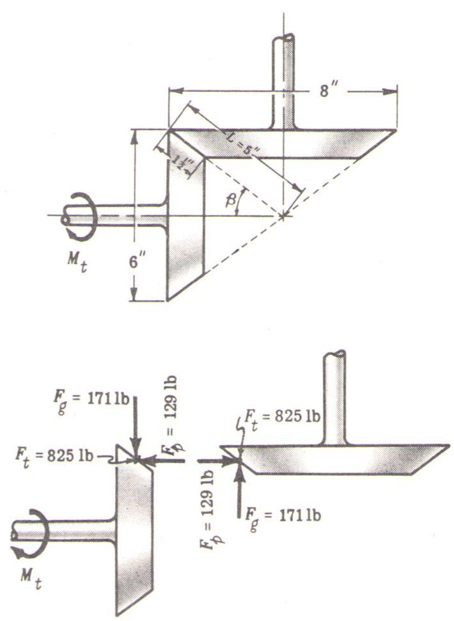


Fig. 17-15

6. A pair of spiral tooth bevel gears has a 4/3 velocity ratio. The pitch diameter of the pinion is 6 in. The face width is $1\frac{1}{2}$ in. The pinion rotates at 240 rpm. The teeth are 5 diametral pitch with a pressure angle ϕ_n of $14\frac{1}{2}°$ measured in a plane normal to a tooth. Eight horsepower is transmitted from the pinion to the gear. The pinion has a right hand spiral and the rotation is clockwise (looking towards the apex of the pitch cone). The spiral angle is $\gamma = 30°$. This corresponds to Case I for spiral bevel gear forces. Determine (*1*) the tangential force F_t at the mean radius, (*2*) the pinion thrust load F_p, (*3*) the gear thrust load F_g.

Refer to Fig. 17-16.

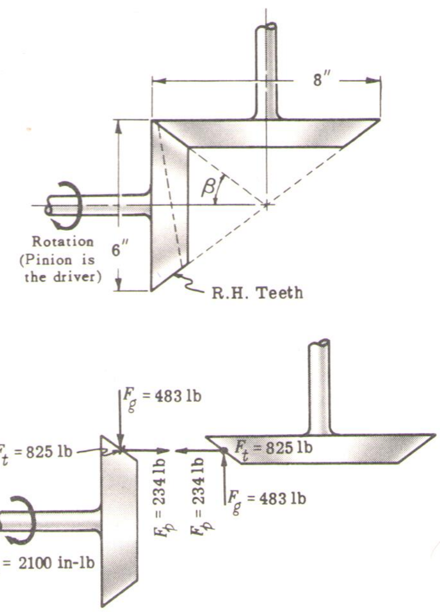


Fig. 17-16

Solution:

(*a*) The values found in Problem 5 that are applicable to this problem are: diameter of gear = 8 in.; slant height of pitch cone = 5 in.; mean radius of pinion = 2.55 in.; pinion torque = 2100 in-lb; tangential force at the mean radius = 825 lb; $\sin\beta = 3/5$, $\cos\beta = 4/5$.

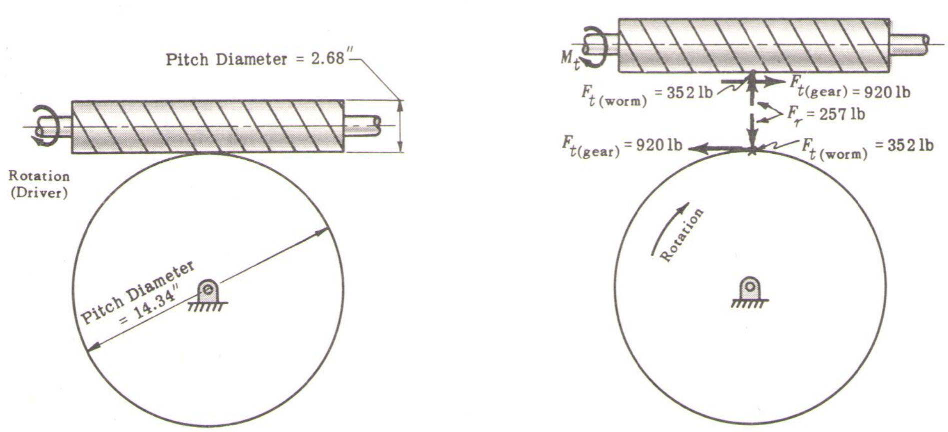
(*b*) Pinion thrust force $F_p = F_t\left(\dfrac{\tan\phi_n \sin\beta}{\cos\gamma} - \tan\gamma\cos\beta\right)$

$$= 825\left[\frac{(\tan 14\frac{1}{2}°)(3/5)}{\cos 30°} - (\tan 30°)(4/5)\right] = -234\,\text{lb}.$$

(*c*) Gear thrust force $F_g = F_t\left(\dfrac{\tan\phi_n\cos\beta}{\cos\gamma} + \tan\gamma\sin\beta\right) = 825\left[\dfrac{(\tan 14\frac{1}{2}°)(4/5)}{\cos 30°} + (\tan 30°)(3/5)\right] = +483\,\text{lb}.$

The force components are shown in the correct directions in the free body diagrams of Fig. 17-16.

7. A worm transmitting 9 hp at 1200 rpm drives a worm gear rotating at 60 rpm. The pitch diameter of the worm is 2.68 in. and the worm is triple threaded. The circular pitch of the worm gear is $\frac{3}{4}$ in. (which is the same as the axial pitch of the worm). The worm gear has 60 teeth which are 20° stub. The coefficient of friction f is 0.10. The worm is right handed and rotates as shown in Fig. 17-17 below. (Note that the figure corresponds to Case I under Worm Gearing. Also note that the power output is not equal to the power input because of friction.) Calculate (*1*) the tangential force $F_{t\,(\text{worm})}$ on the worm, (*2*) the tangential force $F_{t\,(\text{gear})}$ on the gear, (*3*) the separating force F_r.



Solution:

Fig. 17-17

(*a*) Torque on the worm, $M_t = (\text{hp})(63{,}024)/N = 9(63{,}024)/1200 = 472$ in-lb.

(*b*) $F_{t\,(\text{worm})} = M_t/r = 472/(2.68/2) = 352$ lb

(*c*) $$F_{t(\text{gear})} = F_{t(\text{worm})}\frac{1 - f\tan\alpha/\cos\phi_n}{\tan\alpha + f/\cos\phi_n} = 352\left(\frac{1 - 0.1(0.268)/\cos 20^\circ}{0.268 + 0.1/\cos 20^\circ}\right) = 920\text{ lb}$$

where $\tan\alpha = \text{lead}/(\pi D_w) = (3)(\frac{3}{4})/(2.68\pi) = 0.268 \quad (\alpha = 15^\circ)$.

(*d*) $$F_r = F_{t(\text{gear})}\left(\frac{\sin\phi_n}{\cos\phi_n\cos\alpha - f\sin\alpha}\right) = 920\left(\frac{\sin 20^\circ}{\cos 20^\circ\cos 15^\circ - 0.1\sin 15^\circ}\right) = 257\text{ lb}$$

(*e*) Another method of calculating the tangential force on the gear is to employ the efficiency equation for worm gearing to find the output horsepower. The efficiency *e* for worm gearing is

$$e = (\tan\alpha)\left(\frac{\cos\phi_n - f\tan\alpha}{\cos\phi_n\tan\alpha + f}\right) = 0.268\left(\frac{\cos 20^\circ - 0.1\tan 15^\circ}{\cos 20^\circ\tan 15^\circ + 0.1}\right) = 69.8\%$$

$$\text{Gear torque} = (\text{output hp})(63{,}024)/N = (9 \times 0.698)(63{,}024)/60 = 6600\text{ in-lb}$$

$$F_{t(\text{gear})} = M_t/r = 6600/(14.34/2) = 920\text{ lb, as in }(c).$$

8. A differential planetary gear system is shown in Fig. 17-18. Gear B rotates at a constant speed, while the speed of gear H is varied to permit variations in speed of gear I. However, for this problem, consider that all gears rotate at a constant speed. Neglect friction.

Power is applied to gear B and either applied to or taken from gears H and I. Determine how much power must be applied to or taken from gear H if 3 horsepower is supplied to gear B at 1800 rpm of gear B, the direction of rotation of gear B being clockwise as viewed from the right. Gear H is rotating at 700 rpm counterclockwise as viewed from the right. Determine also the power supplied to or taken from gear I and the angular velocity of gear I.

All gears have a pressure angle of $14\frac{1}{2}^\circ$. The diameter of each gear is as follows:

- gear A: 4.0″
- gear B: 5.0″
- gear C: 7.0″
- gear D: 9″(mean dia.)
- gear E: 6.0″(mean dia.)
- gear F: 6.0″(mean dia.)
- gear G: 10.0″(mean dia.)
- gear H: 12.0″
- gear I: 3.0″

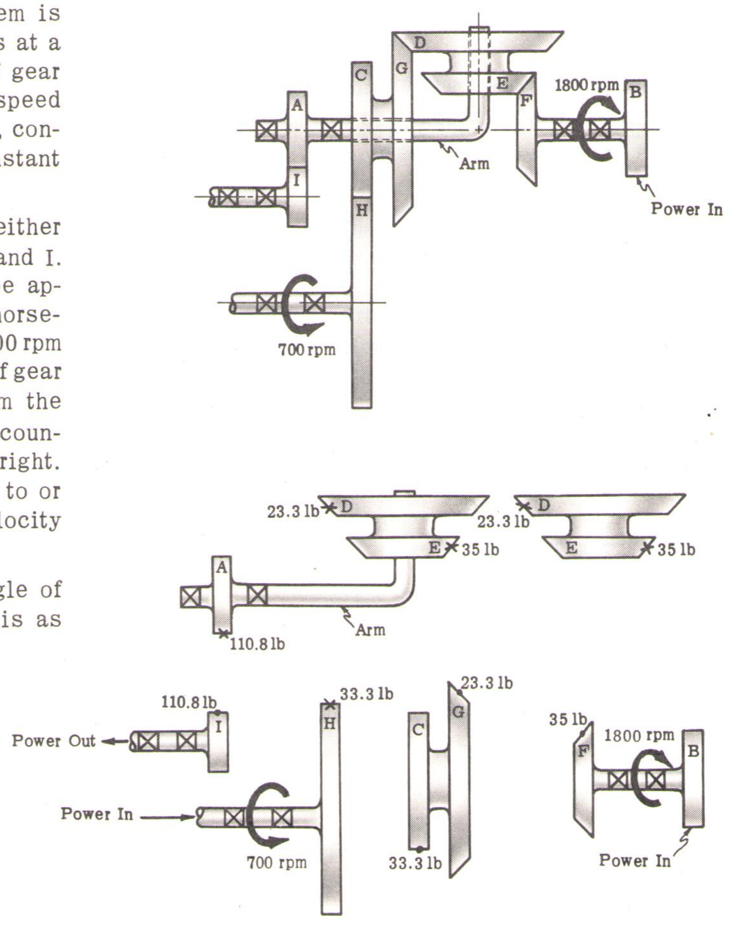


Fig. 17-18

Solution:

(*a*) The forces shown in the free body diagrams are the tangential forces causing torque about the centerline axis of the corresponding gear.