Review and Summary

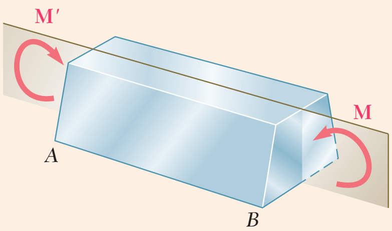

Fig. 4.63 Member in pure-bending.

This chapter was devoted to the analysis of members in *pure bending*. The stresses and deformation in members subjected to equal and opposite couples **M** and **M**′ acting in the same longitudinal plane (Fig. 4.63) were studied.

Normal Strain in Bending

In members possessing a plane of symmetry and subjected to couples acting in that plane, it was proven that *transverse sections remain plane* as a member is deformed. A member in pure bending also has a *neutral surface* along which normal strains and stresses are zero. The longitudinal *normal strain* ϵ_x *varies linearly* with the distance y from the neutral surface:

$$\epsilon_x = -\frac{y}{\rho} \tag{4.8}$$

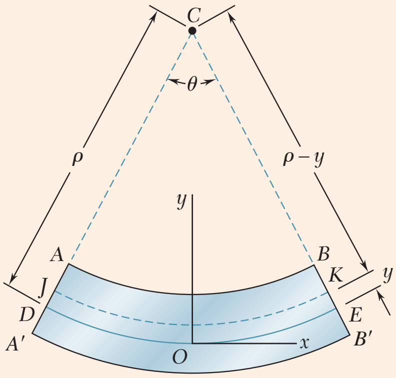

Fig. 4.64 Deformation with respect to neutral axis

where ρ is the *radius of curvature* of the neutral surface (Fig. 4.64). The intersection of the neutral surface with a transverse section is known as the *neutral axis* of the section.

Normal Stress in Elastic Range

For members made of a material that follows Hooke's law, the *normal stress* σ_x *varies linearly* with the distance from the neutral axis (Fig. 4.65). Using the maximum stress σ_m, the normal stress is

$$\sigma_x = -\frac{y}{c}\sigma_m \tag{4.12}$$

where c is the largest distance from the neutral axis to a point in the section.

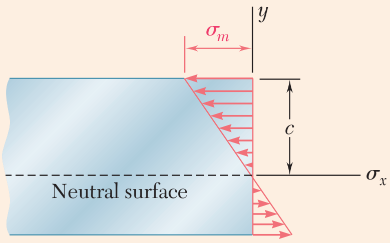

Fig. 4.65 Stress distribution for the elastic flexure formula.

Elastic Flexure Formula

By setting the sum of the elementary forces $\sigma_x\, dA$ equal to zero, we proved that the *neutral axis passes through the centroid* of the cross section of a member in pure bending. Then by setting the sum of the moments of the elementary forces equal to the bending moment, the *elastic flexure formula* is

$$\sigma_m = \frac{Mc}{I} \tag{4.15}$$

where I is the moment of inertia of the cross section with respect to the neutral axis. The normal stress at any distance y from the neutral axis is

$$\sigma_x = -\frac{My}{I} \tag{4.16}$$

Elastic Section Modulus

Noting that I and c depend only on the geometry of the cross section we introduced the *elastic section modulus*

$$S = \frac{I}{c} \tag{4.17}$$

Use the section modulus to write an alternative expression for the maximum normal stress:

$$\sigma_m = \frac{M}{S} \tag{4.18}$$

Curvature of Member

The *curvature* of a member is the reciprocal of its radius of curvature, and may be found by

$$\frac{1}{\rho} = \frac{M}{EI} \tag{4.21}$$

Anticlastic Curvature

In the bending of homogeneous members possessing a plane of symmetry, deformations occur in the plane of a transverse cross section and result in *anticlastic curvature* of the members.

Members Made of Several Materials

We considered the bending of members made of several materials with *different moduli of elasticity*. While transverse sections remain plane, the *neutral axis does not pass through the centroid* of the composite cross section (Fig. 4.66). Using the ratio of the moduli of elasticity of the materials, we obtained a *transformed section* corresponding to an equivalent member made entirely of one material. The methods previously developed are used to determine the stresses in this equivalent homogeneous member (Fig. 4.67), and the ratio of the moduli of elasticity is used to determine the stresses in the composite beam.

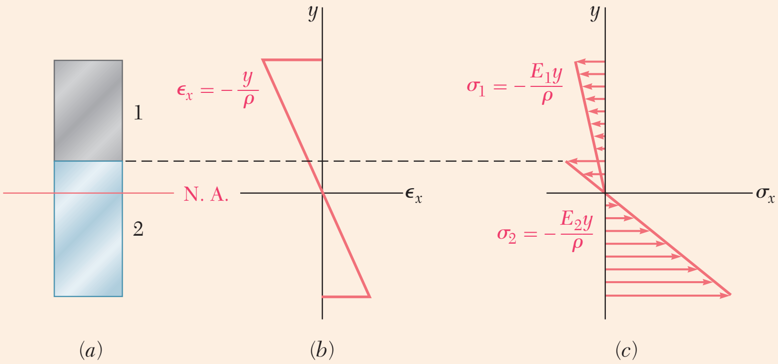

Fig. 4.66 (*a*) Composite section. (*b*) Strain distribution. (*c*) Stress distribution.

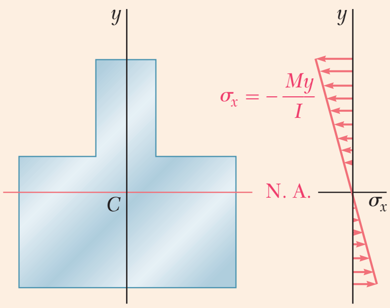

Fig. 4.67 Transformed section.

Stress Concentrations

Stress concentrations occur in members in pure bending and were discussed; charts giving stress-concentration factors for flat bars with fillets and grooves also were presented in Figs. 4.24 and 4.25.

Plastic Deformations

A rectangular beam made of an *elastoplastic material* (Fig. 4.68) was analyzed as the magnitude of the bending moment was increased (Fig. 4.69). The *maximum elastic moment* M_Y occurs when yielding is initiated in the beam (Fig. 4.69*b*). As the bending moment is increased, plastic zones develop (Fig. 4.69*c*), and the size of the elastic core of the member is decreased. When the beam becomes fully plastic (Fig. 4.69*d*), the maximum or *plastic moment* M_p is obtained. *Permanent deformations* and *residual stresses* remain in a member after the loads that caused yielding have been removed.

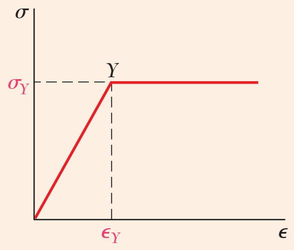

Fig. 4.68 Elastoplastic stress-strain diagram.

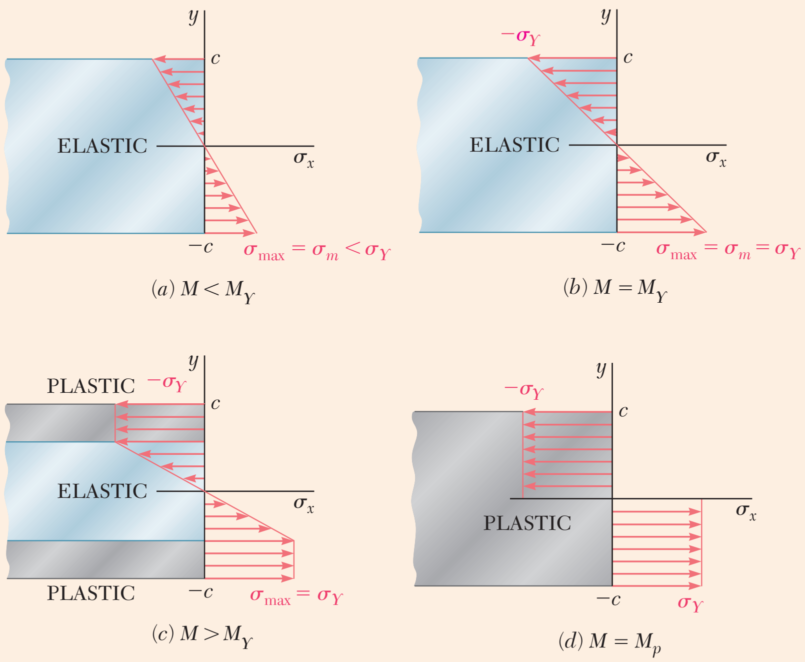

Fig. 4.69 Bending stress distribution in a member for : (*a*) elastic, $M < M_Y$ (*b*) yield impending, $M = M_Y$, (*c*) partially yielded, $M > M_Y$, and (*d*) fully plastic, $M = M_p$.

Eccentric Axial Loading

When a member is loaded *eccentrically in a plane of symmetry*, the *eccentric load* is replaced with a force-couple system located at the centroid of the cross section (Fig. 4.70). The stresses from the centric load and the bending couple are superposed (Fig. 4.71):

$$\sigma_x = \frac{P}{A} - \frac{My}{I} \qquad \textbf{(4.50)}$$

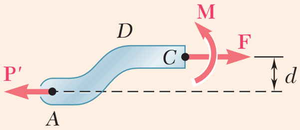

Fig. 4.70 Section of an eccentrically loaded member.

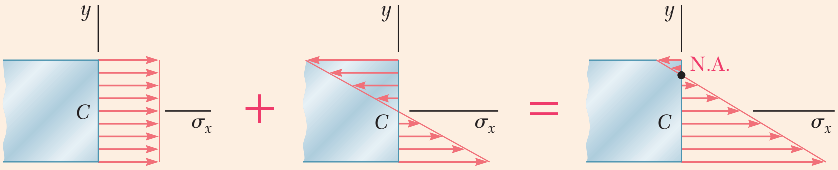

Fig. 4.71 Stress distribution for eccentric loading is obtained by superposing the axial and pure bending distributions.

Unsymmetric Bending

For bending of members of *unsymmetric cross section*, the flexure formula may be used, provided that the couple vector **M** is directed along one of the principal centroidal axes of the cross section. When necessary, **M** can be resolved into components along the principal axes, and the stresses superposed due to the component couples (Figs. 4.72 and 4.73).

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad \textbf{(4.55)}$$

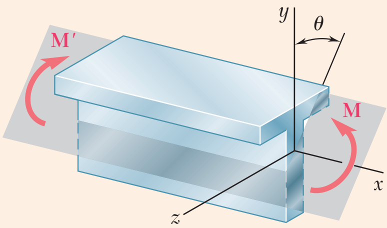

Fig. 4.72 Unsymmetric bending with bending moment not in a plane of symmetry.

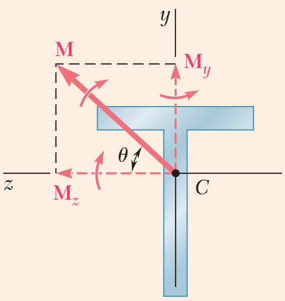

Fig. 4.73 Applied moment resolved into *y* and *z* components.

For the couple **M** shown in Fig. 4.74, the orientation of the neutral axis is defined by

$$\tan \phi = \frac{I_z}{I_y} \tan \theta \qquad \textbf{(4.57)}$$

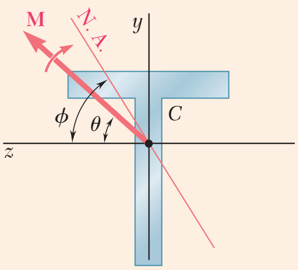

Fig. 4.74 Neutral axis for unsymmetric bending.

General Eccentric Axial Loading

For the general case of *eccentric axial loading*, the load is replaced by a force-couple system located at the centroid. The stresses are superposed due to the centric load and the two component couples directed along the principal axes:

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad \textbf{(4.58)}$$

Curved Members

In the analysis of stresses in *curved members* (Fig. 4.75), transverse sections remain plane when the member is subjected to bending. The *stresses do not vary linearly*, and the neutral surface does not pass through the centroid of the section. The distance R from the center of curvature of the member to the neutral surface is

$$R = \frac{A}{\int \frac{dA}{r}} \qquad \textbf{(4.66)}$$

where A is the area of the cross section. The normal stress at a distance y from the neutral surface is

$$\sigma_x = -\frac{My}{Ae(R-y)} \qquad \textbf{(4.70)}$$

where M is the bending moment and e is the distance from the centroid of the section to the neutral surface.

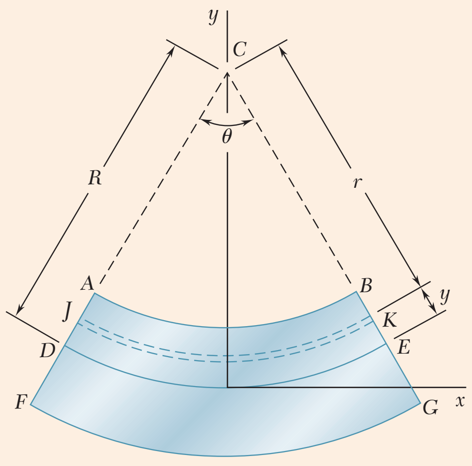

Fig. 4.75 Curved member geometry.