Review and Summary

This chapter was devoted to the concept of stress and to an introduction to the methods used for the analysis and design of machines and load-bearing structures. Emphasis was placed on the use of a *free-body diagram* to obtain equilibrium equations that were solved for unknown reactions. Free-body diagrams were also used to find the internal forces in the various members of a structure.

Axial Loading: Normal Stress

The concept of *stress* was first introduced by considering a two-force member under an *axial loading*. The *normal stress* in that member (Fig. 1.41) was obtained by

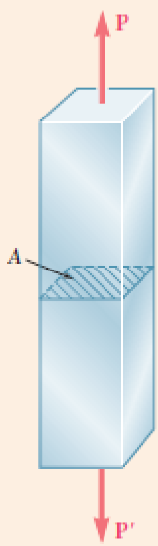

Fig. 1.41 Axially loaded member with cross section normal to member used to define normal stress.

$$\sigma = \frac{P}{A} \tag{1.5}$$

The value of σ obtained from Eq. (1.5) represents the *average stress* over the section rather than the stress at a specific point Q of the section. Considering a small area ΔA surrounding Q and the magnitude ΔF of the force exerted on ΔA, the stress at point Q is

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

In general, the stress σ at point Q in Eq. (1.6) is different from the value of the average stress given by Eq. (1.5) and is found to vary across the section. However, this variation is small in any section away from the points of application of the loads. Therefore, the distribution of the normal stresses in an axially loaded member is assumed to be *uniform*, except in the immediate vicinity of the points of application of the loads.

For the distribution of stresses to be uniform in a given section, the line of action of the loads **P** and **P′** must pass through the centroid C. Such a loading is called a *centric* axial loading. In the case of an *eccentric* axial loading, the distribution of stresses is *not* uniform.

Transverse Forces and Shearing Stress

When equal and opposite *transverse forces* **P** and **P′** of magnitude P are applied to a member AB (Fig. 1.42), *shearing stresses* τ are created over any section located between the points of application of the two forces.

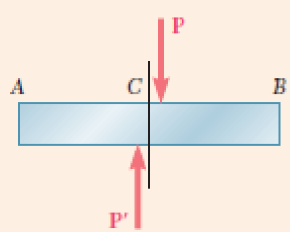

Fig. 1.42 Model of transverse resultant forces on either side of C resulting in shearing stress at section C.

These stresses vary greatly across the section and their distribution *cannot* be assumed to be uniform. However, dividing the magnitude P—referred to as the *shear* in the section—by the cross-sectional area A, the *average shearing stress* is:

$$\tau_{\text{ave}} = \frac{P}{A} \tag{1.8}$$

Single and Double Shear

Shearing stresses are found in bolts, pins, or rivets connecting two structural members or machine components. For example, the shearing stress of bolt CD (Fig. 1.43), which is in *single shear,* is written as

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \tag{1.9}$$

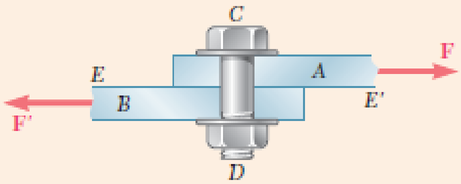

Fig. 1.43 Diagram of a single-shear joint.

The shearing stresses on bolts EG and HJ (Fig. 1.44), which are both in *double shear,* are written as

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

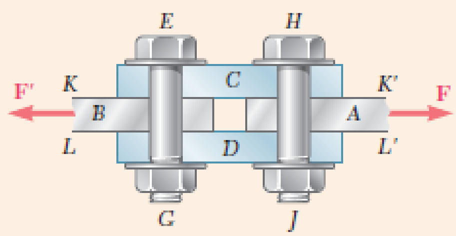

Fig. 1.44 Free-body diagram of a double-shear joint.

Bearing Stress

Bolts, pins, and rivets also create stresses in the members they connect along the *bearing surface* or surface of contact. Bolt CD of Fig. 1.43 creates stresses on the semicylindrical surface of plate A with which it is in contact (Fig. 1.45). Since the distribution of these stresses is quite complicated, one uses an average nominal value σ_b of the stress, called *bearing stress.*

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

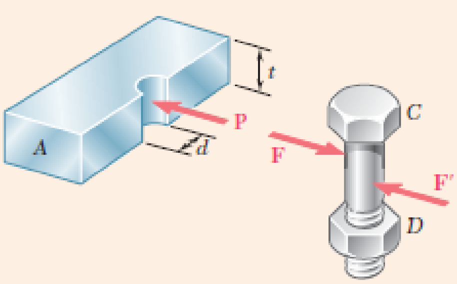

Fig. 1.45 Bearing stress from force P and the single-shear bolt associated with it.

Method of Solution

Your solution should begin with a clear and precise *statement* of the problem. Then draw one or several *free-body diagrams* that will be used to write *equilibrium equations*. These equations will be solved for *unknown forces,* from which the required *stresses* and *deformations* can be computed. Once the answer has been obtained, it should be *carefully checked.*

These guidelines are embodied by the SMART problem-solving methodology, where the steps of Strategy, Modeling, Analysis, and Reflect & Think are used. You are encouraged to apply this SMART methodology in the solution of all problems assigned from this text.

Stresses on an Oblique Section

Fig. 1.46 Axially loaded member with oblique section plane.

When stresses are created on an *oblique section* in a two-force member under axial loading, both *normal* and *shearing* stresses occur. Denoting by θ the angle formed by the section with a normal plane (Fig. 1.46) and by A_0 the area of a section perpendicular to the axis of the member, the normal stress σ and the shearing stress τ on the oblique section are

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta \tag{1.14}$$

We observed from these formulas that the normal stress is maximum and equal to $\sigma_m = P/A_0$ for $\theta = 0$, while the shearing stress is maximum and equal to $\tau_m = P/2A_0$ for $\theta = 45°$. We also noted that $\tau = 0$ when $\theta = 0$, while $\sigma = P/2A_0$ when $\theta = 45°$.

Stress Under General Loading

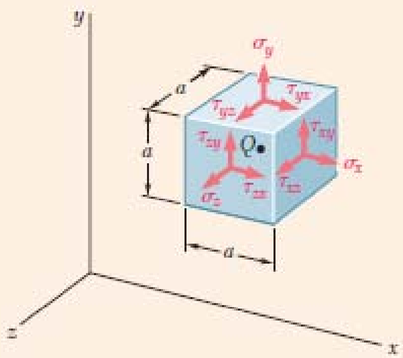

Fig. 1.47 Positive stress components at point *Q*.

Considering a small cube centered at *Q* (Fig. 1.47), σ_x is the normal stress exerted on a face of the cube perpendicular to the *x* axis, and τ_{xy} and τ_{xz} are the *y* and *z* components of the shearing stress exerted on the same face of the cube. Repeating this procedure for the other two faces of the cube and observing that $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, and $\tau_{zx} = \tau_{xz}$, it was determined that *six stress components* are required to define the state of stress at a given point *Q*, being σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}.

Factor of Safety

The *ultimate load* of a given structural member or machine component is the load at which the member or component is expected to fail. This is computed from the *ultimate stress* or *ultimate strength* of the material used. The ultimate load should be considerably larger than the *allowable load* (i.e., the load that the member or component will be allowed to carry under normal conditions). The ratio of the ultimate load to the allowable load is the *factor of safety:*

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate load}}{\text{allowable load}} \tag{1.25}$$

Load and Resistance Factor Design

Load and Resistance Factor Design (*LRFD*) allows the engineer to distinguish between the uncertainties associated with the structure and those associated with the load.