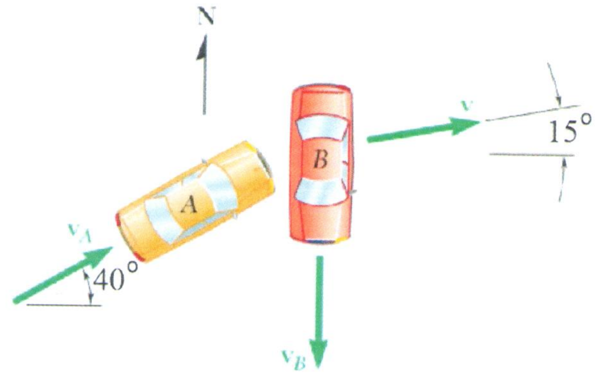
Alabama A&M University

Mechanical Engineering Department

ME 206 – Dynamics: Fall 2016 – Test #2

Student's name KEY

1. At an intersection car B was traveling south and car A was traveling 40° north of east when they slammed into each other. Upon investigation it was found that after the crash two cars got stuck and skidded off at an angle of 15° north of east. Each driver claimed that he was going at the speed limit of 60 km/h and that he tried to slow down but couldn't avoid the crash because the other driver was going a lot faster. Knowing that the masses of cars A and B were 1600 kg and 1200 kg, respectively, determine (a) which car was going faster, (b) the speed of the faster of the two cars if the slower car was traveling at the speed limit.

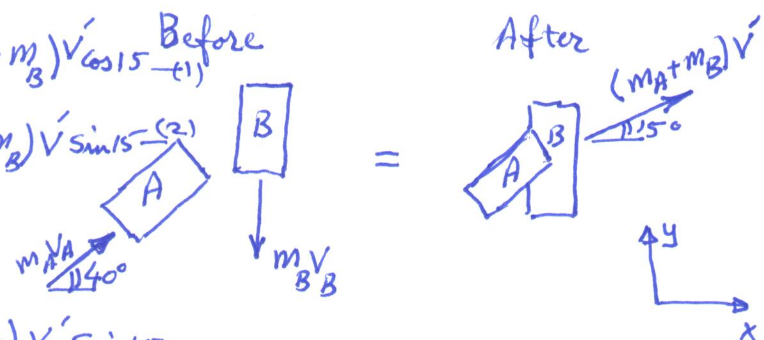


Solution:

a) Total momentum of the two cars is conserved:

$\Sigma mv, x$: $\quad m_A V_A \cos 40 + 0 = (m_A + m_B) V' \cos 15 \quad (1)$

$\Sigma mv, y$: $\quad m_A V_A \sin 40 - m_B V_B = (m_A + m_B) V' \sin 15 \quad (2)$



Dividing eqn (2) by eqn (1)

$$\frac{m_A V_A \sin 40 - m_B V_B}{m_A V_A \cos 40} = \frac{(m_A + m_B) V' \sin 15}{(m_A + m_B) V' \cos 15}$$

$$\tan 40 - \frac{m_B V_B}{m_A V_A \cos 40} = \tan 15$$

$$\tan 40 - \tan 15 = \frac{m_B}{m_A} \cdot \frac{V_B}{V_A} \cdot \frac{1}{\cos 40}$$

$$\frac{V_B}{V_A} = (\tan 40 - \tan 15) \cdot \frac{m_A}{m_B} \cdot \cos 40 = 0.583$$

Hence: $V_B = 0.583\, V_A$

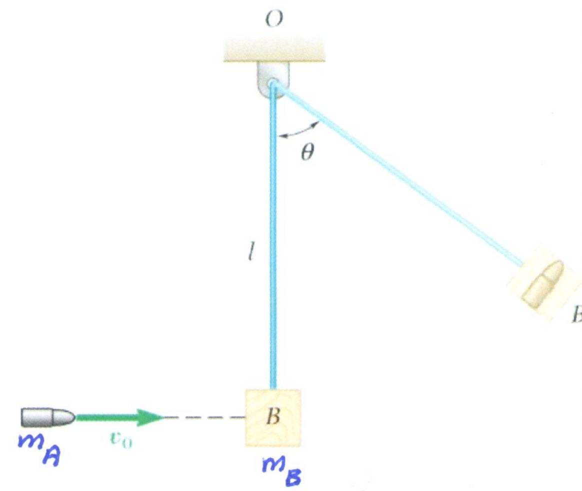
Thus: $\boxed{V_A \text{ was going faster}}$

b) Since V_B was the slower car

$V_B = 60$ km/h

$V_A = \dfrac{V_B}{0.583} = 102.92$ km/h

2. A ballistic pendulum is used to measure the speed of high-speed projectiles. A 9-g bullet A is fired into a 1-kg wood block B suspended by a cord of length l= 2.5 m. The block then swings through a maximum angle of θ= 60°. Determine (a) the initial speed of the bullet v_o, (b) the impulse imparted by the bullet on the block, (c) the force on the cord immediately after the impact.



Given: $m_A = 9\ g = 0.009\ kg.$

$m_B = 1\ kg$

$l = 2.5\ m$

$\theta = 60^o$

Required: a) V_o

b) Impulse momentum

c) T

Solution:

a)

Impulse-Momentum Diagram just before and after Impact.

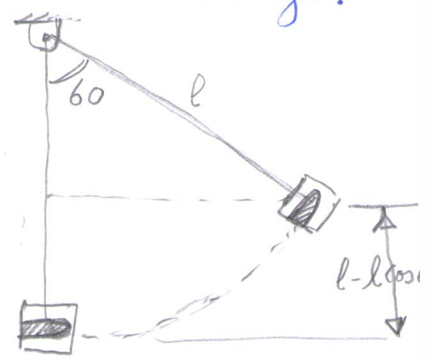
$$\Sigma(mv)_{before} = \Sigma(mv)_{after}$$

$$m_A V_o + m_B \cancel{V_{B_o}}^{0} = (m_A + m_B) V_1$$

$$m_A V_o = (m_A + m_B) V_1 \qquad \text{———— (1)}$$

Conservation of Energy between the Impact location and the max. height.

$$T_1 + \cancel{V_1}^{0} = \cancel{T_2}^{0} + V_2$$



$$\frac{1}{2}\cancel{(m_A + m_B)} V_1^2 = \cancel{(m_A + m_B)}\, g\,(l - l\cos 60)$$

$$V_1^2 = 2gl(1 - \cos 60) = gl$$

$$V_1 = \sqrt{gl} = \sqrt{(9.81)(2.5)} = \boxed{4.95\ m/s}$$

Substituting into eqn(1):

$$V_o = \frac{m_A + m_B}{m_A}(V_1) = \frac{(0.009 + 1)}{0.009}(4.95) = \boxed{554.95\ m/s}$$

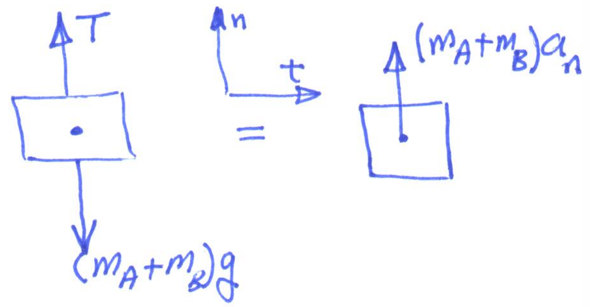
b) Impulse-Momentum of the block only during impact:

$$\cancel{(m_B V)_{before}}^{0} + \underbrace{\int F.dt}_{\text{Impulse}} = m_B V_1$$

Problem 2: Continue:

$$\Sigma \text{Impulse} = \int F \cdot dt = m_B V_1 = (1)(4.95) = \boxed{4.95 \ \ J}$$

c) FBD of Block B just after impact:



n-direction: $\Sigma F_n = m a_n$

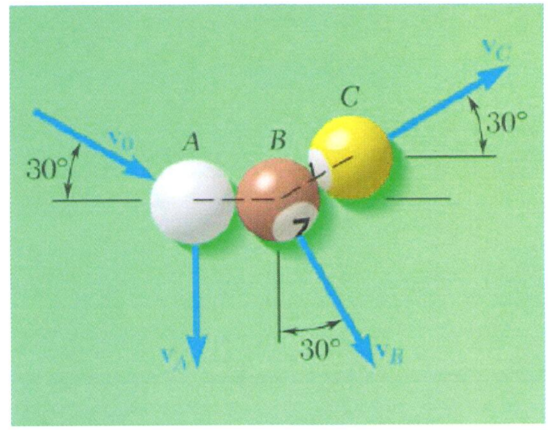
$$T - (m_A + m_B) g = (\ + m_B) a_n$$

$$T = (m_A + m_B) g + (m_A + m_B) \frac{V_1^2}{l}$$

$$= (0.009+1)(9.81) + (0.009+1) \frac{(4.95)^2}{2.5}$$

$$\boxed{T = 19.787 \ \ N}$$

3. In a game of pool, ball A is moving with a velocity v_o of magnitude $v_o = 16$ ft/s when it strikes balls B and C, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (that is, conservation of energy), determine the magnitudes of the velocities v_A, v_B, and v_C.



Solution:

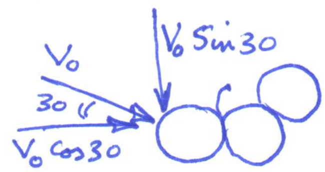
Velocity vectors:

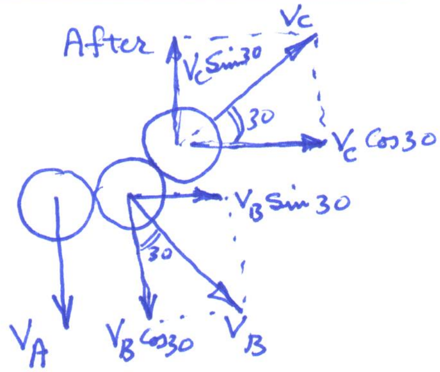
$$\vec{V}_o = V_o \cos 30\, \underline{i} + V_o \sin 30\, j \quad \text{before}$$

$$\vec{V}_A = -V_A\, \underline{j}$$

$$\vec{V}_B = V_B \sin 30\, i - V_B \cos 30\, j$$

$$\vec{V}_C = V_C \cos 30\, i + V_C \sin 30\, j$$





Conservation of momentum:

$$\Sigma(mv)_{before} = \Sigma(mv)_{after}$$

$$m\vec{V}_o = m\vec{V}_A + m\vec{V}_B + m\vec{V}_C$$

Divide by m:

$$\vec{V}_o = \vec{V}_A + \vec{V}_B + \vec{V}_C$$

x-direction: (i):

$$V_o \cos 30 = V_B \sin 30 + V_C \cos 30 \qquad (1)$$

Y-direction: (j):

$$-V_o \sin 30 = -V_A - V_B \cos 30 + V_C \sin 30 \qquad (2)$$

Conservation of Energy: $(\Sigma T)_{before} = (\Sigma T)_{after}$

$$\frac{1}{2} m V_o^2 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 + \frac{1}{2} m_C V_C^2$$

Divide by $\frac{1}{2}$ m:

$$V_o^2 = V_A^2 + V_B^2 + V_C^2 \qquad (3)$$

From equations (1) & (2):

$$V_B = \frac{\sqrt{3}}{2}(V_o - V_A) \quad \& \quad V_C = \frac{1}{2}(V_o + V_A) \qquad (4) \& (5)$$

Substituting into eqn (3):

$$V_o^2 = V_A^2 + \frac{3}{4}(V_o - V_A)^2 + \frac{1}{4}(V_o + V_A)^2$$

$$V_o^2 = 2V_A^2 + V_o^2 - V_o V_A$$

Problem 3: Continue:

$$2V_A^2 = V_o V_A$$

$$V_A = \frac{V_o}{2} = \frac{16}{2} = \boxed{8 \text{ ft/s}}$$

Substituting into (4) and (5):

$$V_B = \frac{\sqrt{3}}{2}(16-8) = \boxed{6.93 \text{ ft/s}}$$

$$V_C = \frac{1}{2}(16+8) = \boxed{12 \text{ ft/s}}$$