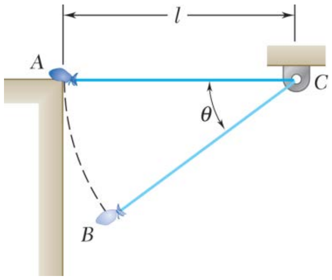


PROBLEM 13.41

A bag is gently pushed off the top of a wall at A and swings in a vertical plane at the end of a rope of length l Determine the angle θ for which the rope will break, knowing that it can withstand a maximum tension equal to twice the weight of the bag.

SOLUTION

Use work - energy: position 1 is at A, position 2 is at B.

$$T_1 + U_{1\to 2} = T_2 \qquad (1)$$

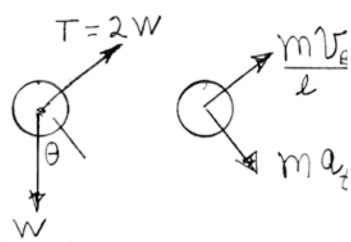
Where $T_1 = 0;\quad U_{1\to 2} = mg\,l\sin\theta;\ \ T_2 = \frac{1}{2}mv_B^2$

Substitute

$$0 + mg\,l\,\sin\theta = \frac{1}{2}mv_B^2$$

$$v_B^2 = 2g\,l\sin\theta \qquad (2)$$

For $T = 2\,W$ use Newton's 2nd law.



$$\nearrow \Sigma F_n = ma_n \Rightarrow 2W - W\sin\theta = \frac{mv_B^2}{l} \qquad (3)$$

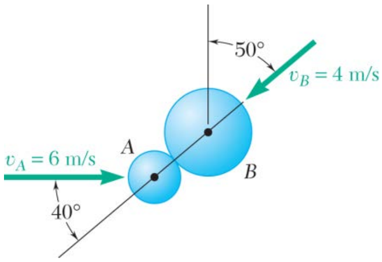
Substitute (2) into (3)

$$2\cancel{m}g - \cancel{m}g\sin\theta = 2\cancel{m}g\frac{\cancel{l}\sin\theta}{\cancel{l}}$$

$$2 = 3\sin\theta$$

$$\text{or } \sin\theta = \frac{2}{3} \Rightarrow \theta = 41.81^\circ$$

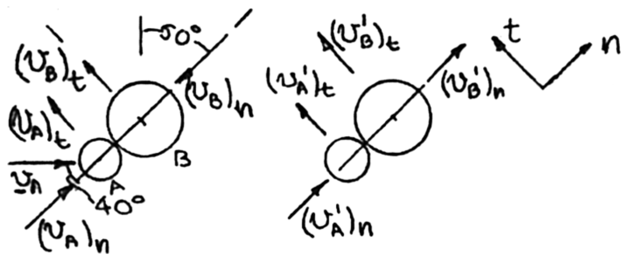
$\theta = 41.8°$ ◀



PROBLEM 13.166

A 600-g ball A is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball B which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact.

SOLUTION



Before   After

$$v_A = 6 \text{ m/s}$$
$$(v_A)_n = (6)(\cos 40°) = 4.596 \text{ m/s}$$
$$(v_A)_t = -6(\sin 40°) = -3.857 \text{ m/s}$$
$$v_B = (v_B)_n = -4 \text{ m/s}$$
$$(v_B)_t = 0$$

t-direction:

Total momentum conserved:

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v'_B)_t + m_B(v'_B)_t$$
$$(0.6 \text{ kg})(-3.857 \text{ m/s}) + 0 = (0.6 \text{ kg})(v'_A)_t + (1 \text{ kg})(v'_B)t$$
$$-2.314 \text{ m/s} = 0.6\,(v'_A)_t + (v'_B)_t \qquad (1)$$

Ball A alone:

Momentum conserved:

$$m_A(v_A)_t = m_A(v'_A)_t \qquad -3.857 = (v'_A)_t$$
$$(v'_A)_t = -3.857 \text{ m/s} \qquad (2)$$

Replacing $(v'_A)_t$ in (2) in Eq. (1)

$$-2.314 = (0.6)(-3.857) + (v'_B)_t$$
$$-2.314 = -2.314 + (v'_B)_t$$
$$(v'_B)_t = 0$$

PROBLEM 13.166 (Continued)

n-direction:

Relative velocities:

$$[(v_A)_n - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$
$$[(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n$$
$$6.877 = (v'_B)_n - (v'_A)_n \qquad (3)$$

Total momentum conserved:

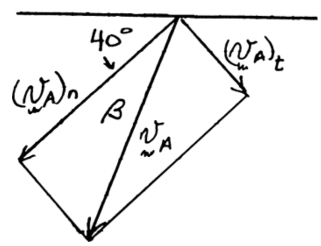
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$
$$(0.6\text{ kg})(4.596\text{ m/s}) + (1\text{ kg})(-4\text{ m/s}) = (1\text{ kg})(v'_B)_n + (0.6\text{ kg})(v'_A)_n$$
$$-1.2424 = (v'_B)_n + 0.6(v'_A)_n \qquad (4)$$

Solving Eqs. (4) and (3) simultaneously,

$$(v'_A)_n = 5.075\text{ m/s}$$
$$(v'_B)_n = 1.802\text{ m/s}$$

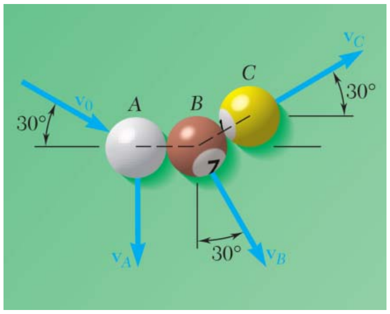
Velocity of *A*:

$$\tan\beta = \frac{|(v_A)_t|}{|(v_A)_n|} = \frac{3.857}{5.075}$$
$$\beta = 37.2° \qquad \beta + 40° = 77.2°$$
$$v'_A = \sqrt{(3.857)^2 + (5.075)^2} = 6.37\text{ m/s}$$



$\mathbf{v}'_A = 6.37\text{ m/s} \measuredangle 77.2°$ ◀

Velocity of *B*: $\mathbf{v}'_B = 1.802\text{ m/s} \measuredangle 40°$ ◀



PROBLEM 14.41

In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 15$ ft/s when it strikes balls B and C, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (that is, conservation of energy), determine the magnitudes of the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$.

SOLUTION

Velocity vectors: $\mathbf{v}_0 = v_0(\cos 30°\,\mathbf{i} - \sin 30°\,\mathbf{j}) \qquad v_0 = 15$ ft/s

$$\mathbf{v}_A = -v_A\mathbf{j}$$

$$\mathbf{v}_B = v_B(\sin 30°\,\mathbf{i} - \cos 30°\,\mathbf{j})$$

$$\mathbf{v}_C = v_C(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$$

Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

Divide by m and resolve into components.

$$\mathbf{i}: \quad v_0 \cos 30° = v_B \sin 30° + v_C \cos 30°$$

$$\mathbf{j}: \quad -v_0 \sin 30° = -v_A - v_B \cos 30° + v_C \sin 30°$$

Solving for v_B and v_C,

$$v_B = \frac{\sqrt{3}}{2}(v_0 - v_A) \qquad v_C = \frac{1}{2}(v_0 + v_A)$$

Conservation of energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$

Divide by $\frac{1}{2}m$ and substitute for v_B and v_C.

$$v_0^2 = v_A^2 + \frac{3}{4}(v_0 - v_A)^2 + \frac{1}{4}(v_0 + v_A)^2$$

$$= 2v_A^2 + v_0^2 - v_0 v_A$$

$$v_A = \frac{1}{2}v_0 = 7.5 \text{ ft/s}$$

$v_A = 7.50$ ft/s ◀

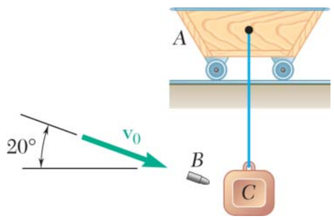
$$v_B = \frac{\sqrt{3}}{2}(15 - 7.5) = 6.4952 \text{ ft/s}$$

$v_B = 6.50$ ft/s ◀

$$v_C = \frac{1}{2}(15 + 7.5) = 11.25 \text{ ft/s}$$

$v_C = 11.25$ ft/s ◀

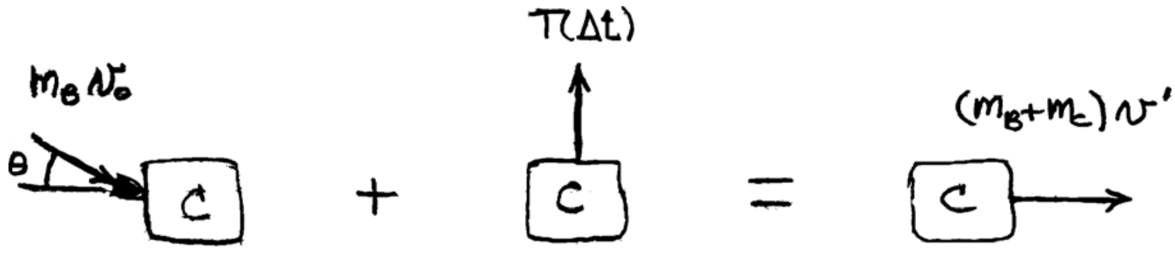
PROBLEM 14.109



Mass C, which has a mass of 4 kg, is suspended from a cord attached to cart A, which has a mass of 5 kg and can roll freely on a frictionless horizontal track. A 60-g bullet is fired with a speed $v_0 = 500$ m/s and gets lodged in block C. Determine (*a*) the velocity of C as it reaches its maximum elevation, (*b*) the maximum vertical distance h through which C will rise.

SOLUTION

Consider the impact as bullet B hits mass C. Apply the principle of impulse-momentum to the two particle system.



$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$

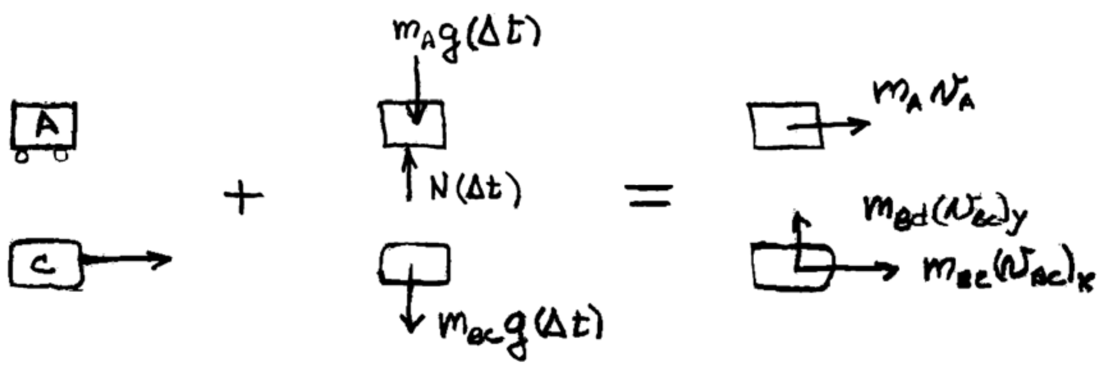
Using both B and C and taking horizontal components gives

$$m_B v_0 \cos\theta + O = (m_B + m_C)v' = m_{BC}v'$$

$$v' = \frac{m_B v_0 \cos\theta}{m_{BC}}$$

$$= \frac{(0.060 \text{ kg})(500 \text{ m/s})\cos 20°}{(4.06 \text{ kg})} = 6.9435 \text{ m/s}$$

Now consider the system of m_A and m_{BC} after the impact, and apply to impulse momentum principle.



PROBLEM 14.109 (Continued)

$$\Sigma m\mathbf{v}_2 + \Sigma \mathbf{Imp}_{2\to 3} = \Sigma m\mathbf{v}_3$$

Horizontal components: $+\longrightarrow$

$$m_{BC}v' + 0 = m_A v_A + m_{BC}v_{cx}$$

$$v_A = \frac{m_{BC}}{m_A}(v' - v_{cx})$$

$$= \frac{4.06}{5}(6.9435 - v_{cx})$$

$$v_A = 5.6381 - 0.812v_{cx} \quad \text{in m/s} \tag{1}$$

(*a*) At maximum elevation.

Both particles have the same velocity, thus

$$v_{cx} = v_A$$
$$v_A = 5.6381 - 0.812\,v_A$$

$$v_A = 3.1115 \text{ m/s}$$

$v_A = 3.11$ m/s ◀

(*b*) Conservation of energy: $T_2 + V_2 = T_3 + V_3$

$$T_2 = \frac{1}{2}m_A(0) + \frac{1}{2}m_{BC}(v')^2$$

$$= \frac{1}{2}(4.06)(6.9435)^2 = 97.871 \text{ J}$$

$$V_2 = 0 \qquad \text{(datum)}$$

$$T_3 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_{BC}(v_{Bx}^2 + v_{By}^2)^2$$

$$= \frac{1}{2}(5)(3.1115)^2 + \frac{1}{2}(4.06)[(3.1115)^2 + 0] = 43.857 \text{ J}$$

$$V_3 = m_{BC}gh = (4.06)(9.81)\,h = 39.829 \text{ h}$$

$$97.871 + 0 = 43.857 + 39.829 \text{ h}$$

$h = 1.356$ m ◀

<u>Another method</u>: We observe that no external horizontal forces are exerted on the system consisting of *A*, *B*, and *C*. Thus the horizontal component of the velocity of the mass center remains constant.

$$m = m_A + m_B + m_C = 5 + 0.06 + 4 = 9.06 \text{ kg}$$

$$\bar{v}_x = \frac{m_B v_0 \cos\theta}{m_A + m_B + m_C} = \frac{(0.060 \text{ kg})(500 \text{ m/s})\cos 20°}{9.06 \text{ kg}} = 3.1115 \text{ m/s}$$

(*a*) At maximum elevation, v_A and v_{BC} are equal.

$$v_A = 3.1115 \text{ m/s}$$

$\mathbf{v}_A = 3.11$ m/s $\longrightarrow$ ◀

PROBLEM 14.109 (Continued)

Immediately after the impact of B on C, the velocity v_A is zero.

$$(m_B + m_C)v' = (m_A + m_B + m_C)\overline{v}_x$$

$$v' = \frac{m_A + m_B + m_C}{m_B + m_C}\overline{v}_x = \frac{9.06}{4.06}(3.1115 \text{ m/s}) = 6.9435 \text{ m/s}$$

(*b*) Principle of work and energy: $T_2 + V_2 = T_3 + V_3$

$T_2, V_2,$ and V_3 are calculated as before.

For T_3 we note that the velocities $\mathbf{v}'_A$ and $\mathbf{v}'_{BC}$ relative to the mass center are zero. Thus, T_3 is given by

$$T_3 = \frac{1}{2}m\overline{\mathbf{v}}^2 = \frac{1}{2}(9.06)(3.1115)^2 = 43.857 \text{ J}$$

As before, h is found to be $h = 1.356$ m ◀