PROBLEM 13.41

A bag is gently pushed off the top of a wall at $A$ and swings in a vertical plane at the end of a rope of length $l$. Determine the angle $\theta$ for which the rope will break, knowing that it can withstand a maximum tension equal to twice the weight of the bag.

SOLUTION

Use work - energy: position 1 is at $A$, position 2 is at $B$.

$$ T_1 + U_{1 \rightarrow 2} = T_2 $$

(1)

Where $T_1 = 0$; $U_{1 \rightarrow 2} = mg l \sin \theta$; $T_2 = \frac{1}{2}mv_b^2$

Substitute

$$ 0 + mg l \sin \theta = \frac{1}{2}mv_b^2 $$

$$ v_b^2 = 2gl \sin \theta $$

(2)

For $T = 2W$ use Newton’s 2nd law.

$$ \sum F_n = ma \Rightarrow 2W - W \sin \theta = \frac{mv_b^2}{l} $$

(3)

Substitute (2) into (3)

$$ 2mg - mg \sin \theta = 2mg \frac{2 \sin \theta}{l} $$

$$ 2 = 3 \sin \theta $$

or $\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.81^\circ$

$\theta = 41.8^\circ$
PROBLEM 13.166

A 600-g ball $A$ is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball $B$ which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact.

SOLUTION

$\begin{array}{ll}
\text{Before} & \text{After} \\
6 \text{ m/s} & (6)(\cos 40^\circ) = 4.596 \text{ m/s} \\
(6)(\sin 40^\circ) = 3.857 \text{ m/s} & (v_A)_t = -6(\sin 40^\circ) = -3.857 \text{ m/s} \\
4 \text{ m/s} & (v_B)_t = -4 \text{ m/s} \\
0 & (v_B)_t = 0 \\
\end{array}$

$t$-direction:

Total momentum conserved:

$m_A(v_A)_t + m_B(v_B)_t = m_A(v'_A)_t + m_B(v'_B)_t$

$(0.6 \text{ kg})(-3.857 \text{ m/s}) + 0 = (0.6 \text{ kg})(v'_A)_t + (1 \text{ kg})(v'_B)_t$

$-2.314 \text{ m/s} = 0.6 (v'_A)_t + (v'_B)_t \hspace{1cm} (1)$

Ball $A$ alone:

Momentum conserved:

$m_A(v_A)_t = m_A(v'_A)_t - 3.857 = (v'_A)_t$

$(v'_A)_t = -3.857 \text{ m/s} \hspace{1cm} (2)$

Replacing $(v'_A)_t$ in (2) in Eq. (1)

$-2.314 = 0.6(-3.857) + (v'_B)_t$

$-2.314 = -2.314 + (v'_B)_t$

$(v'_B)_t = 0$
n-direction:

Relative velocities:

\[ [(v_A)_n - (v_B)_n] \cdot e = (v'_A)_n - (v'_B)_n \]
\[ [(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n \]
\[ 6.877 = (v'_B)_n - (v'_A)_n \]  

(3)

Total momentum conserved:

\[ m_A (v_A)_n + m_B (v_B)_n = m_A (v'_A)_n + m_B (v'_B)_n \]
\[ (0.6 \text{ kg})(4.596 \text{ m/s}) + (1 \text{ kg})(-4 \text{ m/s}) = (1 \text{ kg})(v'_B)_n + (0.6 \text{ kg})(v'_A)_n \]
\[ -1.2424 = (v'_B)_n + 0.6(v'_A)_n \]  

(4)

Solving Eqs. (4) and (3) simultaneously,

\[ (v'_A)_n = 5.075 \text{ m/s} \]
\[ (v'_B)_n = 1.802 \text{ m/s} \]

Velocity of A:

\[ \tan \beta = \frac{|(v_A)_n|}{|(v_A)'_n|} \]
\[ = \frac{3.857}{5.075} \]
\[ \beta = 37.2^\circ \]

\[ \beta + 40^\circ = 77.2^\circ \]

\[ (v'_A)_n = \sqrt{(3.857)^2 + (5.075)^2} \]
\[ = 6.37 \text{ m/s} \]

Velocity of B:

\[ v'_B = 1.802 \text{ m/s} \angle 40^\circ \]

\[ v'_A = 6.37 \text{ m/s} \angle 77.2^\circ \]
PROBLEM 14.41

In a game of pool, ball \( A \) is moving with a velocity \( v_0 \) of magnitude \( v_0 = 15 \text{ ft/s} \) when it strikes balls \( B \) and \( C \), which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (that is, conservation of energy), determine the magnitudes of the velocities \( v_A \), \( v_B \), and \( v_C \).

SOLUTION

Velocity vectors:

\[ v_0 = v_0 (\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j}) \quad v_0 = 15 \text{ ft/s} \]

\[ v_A = -v_A \mathbf{j} \]

\[ v_B = v_B (\sin 30^\circ \mathbf{i} - \cos 30^\circ \mathbf{j}) \]

\[ v_C = v_C (\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) \]

Conservation of momentum:

\[ m v_0 = m v_A + m v_B + m v_C \]

Divide by \( m \) and resolve into components.

\[ \mathbf{i}: \quad v_0 \cos 30^\circ = v_B \sin 30^\circ + v_C \cos 30^\circ \]

\[ \mathbf{j}: \quad -v_0 \sin 30^\circ = -v_A - v_B \cos 30^\circ + v_C \sin 30^\circ \]

Solving for \( v_B \) and \( v_C \),

\[ v_B = \frac{\sqrt{3}}{2} (v_0 - v_A) \quad v_C = \frac{1}{2} (v_0 + v_A) \]

Conservation of energy:

\[ \frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 \]

Divide by \( \frac{1}{2} m \) and substitute for \( v_B \) and \( v_C \).

\[ v_0^2 = v_A^2 + \frac{3}{4} (v_0 - v_A)^2 + \frac{1}{4} (v_0 + v_A)^2 \]

\[ = 2v_A^2 + v_0^2 - v_0 v_A \]

\[ v_A = \frac{1}{2} v_0 = 7.5 \text{ ft/s} \quad v_A = 7.50 \text{ ft/s} \]

\[ v_B = \frac{\sqrt{3}}{2} (15 - 7.5) = 6.4952 \text{ ft/s} \quad v_B = 6.50 \text{ ft/s} \]

\[ v_C = \frac{1}{2} (15 + 7.5) = 11.25 \text{ ft/s} \quad v_C = 11.25 \text{ ft/s} \]
PROBLEM 14.109

Mass \(C\), which has a mass of 4 kg, is suspended from a cord attached to cart \(A\), which has a mass of 5 kg and can roll freely on a frictionless horizontal track. A 60-g bullet is fired with a speed \(v_0 = 500\) m/s and gets lodged in block \(C\). Determine (a) the velocity of \(C\) as it reaches its maximum elevation, (b) the maximum vertical distance \(h\) through which \(C\) will rise.

SOLUTION

Consider the impact as bullet \(B\) hits mass \(C\). Apply the principle of impulse-momentum to the two particle system.

\[
\Sigma m v_1 + \Sigma \text{Imp}_{1 \rightarrow 2} = \Sigma m v_2
\]

Using both \(B\) and \(C\) and taking horizontal components gives

\[
m_B v_0 \cos \theta + O = (m_B + m_C) v' = m_{BC} v'
\]

\[
v' = \frac{m_B v_0 \cos \theta}{m_{BC}}
\]

\[
= \frac{(0.060 \text{ kg})(500 \text{ m/s}) \cos 20^\circ}{4.06 \text{ kg}} = 6.9435 \text{ m/s}
\]

Now consider the system of \(m_A\) and \(m_{BC}\) after the impact, and apply to impulse momentum principle.
PROBLEM 14.109 (Continued)

\[ \Sigma m v_2 + \Sigma \text{Imp}_{2 \rightarrow 3} = \Sigma m v_3 \]

Horizontal components:

\[ m_{BC} v_c' + 0 = m_A v_A + m_{BC} v_{cx} \]

\[ v_A = \frac{m_{BC}}{m_A} (v' - v_{cx}) \]

\[ = \frac{4.06}{5} (6.9435 - v_{cx}) \]

\[ v_A = 5.638 - 0.812 v_{cx} \text{ in m/s} \quad (1) \]

(a) At maximum elevation.

Both particles have the same velocity, thus

\[ v_{cx} = v_A \]
\[ v_A = 5.638 - 0.812 v_A \]
\[ v_A = 3.1115 \text{ m/s} \]

\[ v_A = 3.11 \text{ m/s} \]

(b) Conservation of energy:

\[ T_2 + V_2 = T_3 + V_3 \]

\[ T_2 = \frac{1}{2} m_A (0) + \frac{1}{2} m_{BC} (v')^2 \]

\[ = \frac{1}{2} (4.06)(6.9435)^2 = 97.871 \text{ J} \]

\[ V_2 = 0 \quad \text{(datum)} \]

\[ T_3 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_{BC} (v_{cx}^2 + v_{cy}^2)^2 \]

\[ = \frac{1}{2} (5)(3.1115)^2 + \frac{1}{2} (4.06)((3.1115)^2 + 0) = 43.857 \text{ J} \]

\[ V_3 = m_{BC} gh = (4.06)(9.81)(h) = 39.829 \text{ h} \]

\[ 97.871 + 0 = 43.857 + 39.829 \text{ h} \]

\[ h = 1.356 \text{ m} \]

Another method: We observe that no external horizontal forces are exerted on the system consisting of A, B, and C. Thus the horizontal component of the velocity of the mass center remains constant.

\[ m = m_A + m_B + m_C = 5 + 0.06 + 4 = 9.06 \text{ kg} \]

\[ \bar{v}_s = \frac{m_B v_0 \cos \theta}{m_A + m_B + m_C} = \frac{(0.060 \text{ kg})(500 \text{ m/s}) \cos 20^\circ}{9.06 \text{ kg}} = 3.1115 \text{ m/s} \]

(a) At maximum elevation, \( v_A \) and \( v_{BC} \) are equal.

\[ v_A = 3.1115 \text{ m/s} \]

\[ v_A = 3.11 \text{ m/s} \]
PROBLEM 14.109  (Continued)

Immediately after the impact of B on C, the velocity \( v_A \) is zero.

\[
(m_B + m_C)v' = (m_A + m_B + m_C)v_x
\]

\[
v' = \frac{m_A + m_B + m_C}{m_B + m_C} \bar{v}_x = \frac{9.06}{4.06} (3.1115 \text{ m/s}) = 6.9435 \text{ m/s}
\]

(b) Principle of work and energy: \( T_2 + V_2 = T_3 + V_3 \)

\( T_2, V_2, \) and \( V_3 \) are calculated as before.

For \( T_3 \) we note that the velocities \( v'_A \) and \( v'_BC \) relative to the mass center are zero. Thus, \( T_3 \) is given by

\[
T_3 = \frac{1}{2} m \bar{v}^2 = \frac{1}{2} (9.06)(3.1115)^2 = 43.857 \text{ J}
\]

As before, \( h \) is found to be \( h = 1.356 \text{ m} \)